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RADIATIVE INSTABILITY IN STELLAR ENVELOPES

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ABSTRACT

A general analytic criterion for radiative instability in the envelope of a nonexplosive star is derived, based on a generalization of the Eddington limit for the star's luminosity, an expression for the stationary mass-loss acceleration, and an assumed universality of the critical mean density of the envelope. The derived criterion has been formulated as the minimum rate of mass loss needed to achieve a super-Eddington state. Expressed as a simple function of luminosity and effective temperature, it is applied to luminous blue variables (LBVs), Wolf-Rayet stars, and ordinary B-type supergiants. Assuming that the brightest LBVs as well as most of the Wolf-Rayet stars are post–main-sequence objects, these objects must have radiatively unstable envelopes.

Subject headings: stars: interiors — stars: mass loss — stars: variables: other — stars: Wolf-Rayet — supergiants

1. INTRODUCTION

Radiative instability in a nonexplosive star is usually associated with its thin atmospheric layers (e.g., de Jager 1980, chap. 1; de Jager et al. 2001). This instability develops when the radiation pressure force exceeds the inward pull of gravity, and therefore it tends to occur most easily in the loosely bound atmospheric layers of very luminous supergiants, either hot or cool. There has long been a suspicion that it can also occur deeper inside the envelope (Stothers & Chin 1983), but a workable mechanism for it was unavailable until a "bump" in the iron opacity curve at a temperature of $\sim 1.5 \times 10^5$ K showed up in improved opacity calculations made by Iglesias, Rogers, & Wilson (1987). Using purely radiative envelope models, Kato & Iben (1992) demonstrated that this opacity feature could possibly destabilize Wolf-Rayet stars and drive their high rates of mass loss. A counterargument, however, was that turbulent convection might be able to transport the excess luminous flux arising from the opacity bump and so prevent radiative instability from developing anywhere in the interior (Schaerer 1996).

More recently, simple convective mixing-length calculations made very close to the limit of radiative stability have suggested that turbulent convection, even when supersonic, cannot carry all of the excess flux (Stothers 2002). This conclusion was found to be true also for the brightest and hottest luminous blue variables (LBVs or S Doradus variables) if they, like the Wolf-Rayet stars, are post–main-sequence objects. These calculations, however, were performed only for a small number of stellar envelope models, clustered in a very narrow range of effective temperature, $T_e = 20{,}000$–$30{,}000$ K, in which the models were simultaneously crossing the thresholds of radiative instability and ionization-induced dynamical instability. Since the primary focus of that work was dynamical instability, no adequate survey of radiative instability in the envelope has yet been conducted.

In the course of doing a more extensive model survey, a simple criterion for radiative instability has been serendipitously discovered. It is based on Eddington's (1921) limit on the luminosity, along with additional stellar envelope physics that permits an analytic formulation of the criterion in terms of luminosity, effective temperature, and mass-loss rate. Thus, it can be applied to a wide range of luminous stars.

2. GENERALIZED RADIATIVE INSTABILITY

The envelope of a post–main-sequence star with a very high ratio of luminosity to mass (L/M) contains a negligible amount of matter lying above the iron convection zone and, in fact, is so tenuous that above this zone, radiative equilibrium applies to a very good approximation. Ignoring turbulent pressure and axial rotation as being unimportant, the forces acting arise primarily from gravity, gas and radiation pressure, and mass loss (Stothers 1999, 2002, 2003). If the ratio of gas pressure to the sum of gas and radiation pressure is denoted by β, then at radial distance r,

$$1 - \beta = L/L_{\rm E}, \tag{1}$$

where the generalized Eddington luminosity is defined by

$$L_{\rm E} = \frac{4\pi cGM(1-\psi)}{\langle\kappa\rangle}. \tag{2}$$

Here $\langle\kappa\rangle$ represents a running mean opacity, given by

$$\frac{1}{\langle\kappa\rangle} = \frac{1}{P_{\rm rad}} \int_R^r \frac{1}{\kappa} \frac{dP_{\rm rad}}{dr}\, dr, \tag{3}$$

and ψ is a (constant) ratio of mass-loss acceleration to gravitational acceleration, taken to be

$$\psi = \left(\frac{h}{\delta M}\frac{dM}{dt}\right)^2 \frac{R^3}{GM}. \tag{4}$$

In this expression for ψ, δM is the mass of the moving stellar envelope and h is the ratio of the envelope's dynamical response time to its free-fall time. Numerical hydrodynamical models of stellar envelopes with a very large L/M ratio suggest $h = 10$, to within a factor of 2 (Stothers 2002).

Physically, it is impossible that $\beta < 0$. Therefore, equation (1) imposes an upper limit on the luminosity, $L_{\rm E}$. Strictly speaking, $L_{\rm E}$ is a nonlocal function of the radial distance r. However, in the tenuous envelope of an LBV, the basic structure is relatively simple. With β very small everywhere and κ nearly constant (close to the electron-scattering limit), $L/L_{\rm E}$ varies only a little through the outer envelope. Nevertheless, radiative stability vanishes first in the slightly nonadiabatic upper layers of

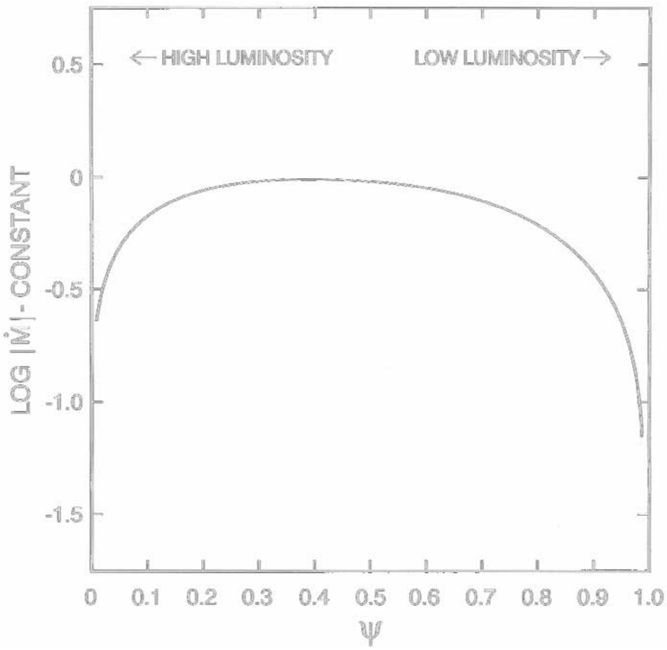

FIG. 1.—Mass-loss rate, normalized to its maximum value, as a function of the dimensionless parameter ψ.

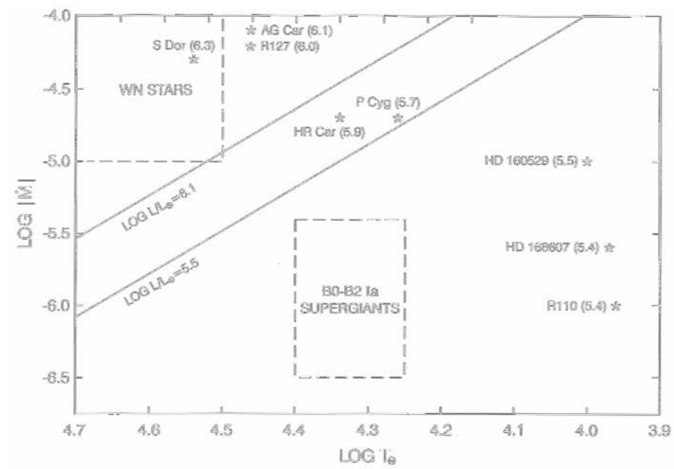

FIG. 2.—Mass-loss rate ($M_\odot$ yr^{-1}) vs. effective temperature (K). Theoretical thresholds of radiative instability in the envelope are plotted in the case of two values of log ($L/L_\odot$); stellar envelopes lying above the theoretical lines are predicted to be radiatively unstable. Asterisks denote well-observed LBVs and LBV candidates at quiescence; observed values of log ($L/L_\odot$) are indicated in parentheses. The approximate domains of the cooler WN stars and of the ordinary B0–B2 Ia supergiants are also plotted.

the iron convection zone; therefore, for δM, we adopt the cumulative mass of the overlying layers.

3. MASS LOSS AT THE EDDINGTON LIMIT

The problem now is to compute, in an approximate but general way, the rate of mass loss when the star lies precisely at the Eddington limit $L = L_E$. The necessary equations consist of equations (2) and (4) with $\delta M = (4/3)\pi R^3 \langle\rho\rangle$. Combining these equations with the Stefan-Boltzmann law, $L = 4\pi R^2 (ac/4) T_e^4$, we get

$$|\dot{M}| = \frac{2^{7/2}\pi (GM)^{5/4} \langle\rho\rangle (1-\psi)^{3/4} \psi^{1/2}}{3h(a\langle\kappa\rangle)^{3/4} T_e^3}. \qquad (5)$$

Observe that $|\dot{M}|$ as a function of ψ is zero at both $\psi = 0$ and $\psi = 1$. The function rises to a single maximum at $\psi = 2/5$. A plot of $|\dot{M}|$, normalized to its maximum value, is displayed in Figure 1. Over the whole range of ψ (except near the two endpoints), $|\dot{M}|$ is roughly constant. Therefore, according to equation (2), $|\dot{M}|$ varies only slightly with luminosity at a given stellar mass. It must be emphasized that $|\dot{M}|$ represents the *minimum* rate of mass loss that is necessary to bring the star into a state of radiative instability.

To use equation (5) more widely, it would be of great advantage if $\langle\kappa\rangle$ and $\langle\rho\rangle$ were approximately constant quantities. A large number of outer envelope models close to the Eddington limit, computed both in our earlier study and over a much expanded range of effective temperature here, show $\langle\kappa\rangle = 0.35$ cm^2 g^{-1} (to within a factor of 1.2) and $\langle\rho\rangle = 2 \times 10^{-11}$ g cm^{-3} (to within a factor of 2). The theoretical uncertainty in $|\dot{M}|$ arising from possible errors generated by our adoption of mean values of $\langle\kappa\rangle$, $\langle\rho\rangle$, and h probably does not exceed a factor of ~3, which is about the same as the estimated error of the observed mass-loss rates for LBVs (Vink & de Koter 2002) and for their close relatives, the hydrogen-poor Wolf-Rayet stars (Hamann & Koesterke 2000).

The (M, L)-relation for LBV models at both the start and the end of central helium burning can now be used to evaluate the threshold value of $|\dot{M}|$ for various effective temperatures. Since the physically valid range of luminosities for each stellar mass (or conversely, the physically valid range of stellar masses for each luminosity) is so small, $|\dot{M}|$ ought to be tightly constrained in the mass coordinate for any luminosity. In view of equation (5) and Figure 1, $|\dot{M}|$ depends, therefore, primarily on effective temperature, $|\dot{M}| \propto T_e^{-3}$, as exhibited in Figure 2. Inspection shows that the approximate luminosity dependence is $|\dot{M}| \propto L$. Therefore,

$$\log |\dot{M}| \approx \log (L/L_\odot) - 3 \log T_e + 2.5. \qquad (6)$$

If a star lies above the line $L = L_E$, the threshold rate of mass loss is being exceeded and the outer envelope must be radiatively unstable. In that case, an amount of mass greater than δM will be ejected during the envelope's dynamical response time. Supersonic turbulence in the iron convection zone may power the rapid mass loss.

4. COMPARISON WITH OBSERVATIONS

On Figure 2 are plotted the empirical mass-loss rates and effective temperatures for eight well-observed LBVs at quiescence tabulated previously (Stothers 2002); the original data come mostly from Leitherer (1997) and van Genderen (2001) but do not include the questionable mass-loss rate for R71, as discussed in our earlier paper. Each LBV in the figure is tagged with its log ($L/L_\odot$) value.

Clearly, the three coolest and faintest LBVs have mass-loss rates that are too small to produce radiative instability in the envelope, but the three hottest and brightest objects are conspicuously unstable. These conclusions seem now to be robust against any reasonable uncertainties about the stars' effective temperatures and luminosities. In order to negate these conclusions, either the empirical or the theoretical mass-loss rates would have to be in error by 1–2 orders of magnitude.

General locations on Figure 2 are indicated also for the coolest Wolf-Rayet stars (Hamann & Koesterke 1998, 2000; Nugis, Crowther, & Willis 1998; Nugis & Lamers 2000) and for the brightest ordinary early B supergiants (Leitherer, Chapman, & Koribalski 1995; Scuderi et al. 1998; Kudritzki et al. 1999; Lamers et al. 1999; Benaglia, Cappa, & Koribalski 2001). Without much doubt, the envelopes of these stars are radiatively unstable and stable, respectively, barring errors of 1–2 orders of magnitude in the empirical or theoretical mass-loss rates.

Our conclusions about the radiative stability of the coolest LBVs and of the ordinary B-type supergiants are actually stronger than indicated. Any comparison of the observed and critical mass-loss rates should in principle be the same as comparing ψ in an actual stellar envelope with the ψ that would be needed to achieve $L = L_{\rm E}$ in that envelope. Accordingly, the values of h and of δM appearing in equation (4) ought to be those prevailing in the actual envelope, not in some extreme case of the envelope with $L = L_{\rm E}$. Our use of such a marginally unstable envelope has been dictated here by a desire for universality, simplicity, and accuracy near the limit $L = L_{\rm E}$. Because $h < 10$ and $\langle\rho\rangle > 2 \times 10^{-11}$ g cm^{-3} if $L < L_{\rm E}$, our predicted critical values of $|\dot{M}|$ for stars lying below the line $L = L_{\rm E}$ in Figure 2 are somewhat too small.

If the brightest LBVs actually are rapidly rotating stars near the end of the main-sequence phase, as some authors believe (e.g., Maeder & Meynet 2000; Lamers et al. 2001), a rotational term should be included in equation (2). In that case, the value of $|\dot{M}|$ needed to achieve radiative instability would be lowered, and the instability could arise primarily from the fast rotation.

Lastly, we note that Figure 2 applies only to radiative instability inside the stellar envelope. Our results do not refer to the stellar atmosphere, where radiative instability can be achieved through strongly turbulent atmospheric motions in the case of luminous supergiants hotter than 8000 K (de Jager et al. 2001).

5. CONCLUSION

The traditional method to test for radiative instability in the stellar interior is to compute $L_{\rm E}$ and then to compare it with L, or, equivalently, to solve the equation $L = L_{\rm E}$ for the critical mass M in order to compare it with the observed (or theoretically predicted) mass. In such a conventional test, the mass-loss acceleration is traditionally ignored. Quite apart from the fact that the test thereby becomes inaccurate, the demands placed on an accurate knowledge of both L and M make the conventional test unreliable in any case.

When the mass-loss acceleration is taken into account, it yields an additional degree of freedom, so that one can in practice compare the observed mass-loss rate of a star with the value required for radiative instability of its envelope. Only rough information about the star's mass or luminosity need be known.

It has been shown here that a very simple formula links $|\dot{M}|$, L, and T_e at the threshold of radiative instability. Applied to observed stars, the criterion predicts that the brightest LBVs as well as the coolest WN stars are radiatively unstable, on the assumption that these objects are evolving in a post–main-sequence phase. Because other massive, evolved Wolf-Rayet stars also possess high values of $|\dot{M}|$, L, and T_e, it is predicted that they, too, are radiatively unstable.

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